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Summary

The objective of this work package was the development of the required theory that will form the basis for the succeeding Work Packages. In Deliverable 7 (D7) advanced network theory was used in order to formulate the necessary mathematical expressions (transfer functions) that will enable the customization of the location and the width of the four selected bands of operation. Periodic analysis was used to describe the dispersion characteristics of the Generalized Negative Refractive Index Transmission Line (NRI-TL). To improve design accuracy the unit cell was extended to contain the effect of the host transmission lines that are unavoidable in a practical circuit. The new dispersion relation was derived from the periodic analysis and a MATLAB function was created to yield the resulting frequencies for a desired phase shift. The discrete component values (capacitors and inductors) can also be calculated using the given design frequencies and proper phase shifts for each type of unit cell required for the RF blocks that make up the receiver chain.

Introduction

GNRI-TL Theory

Fig. 1 shows the model of a typical Generalized NRI-TL unit cell .

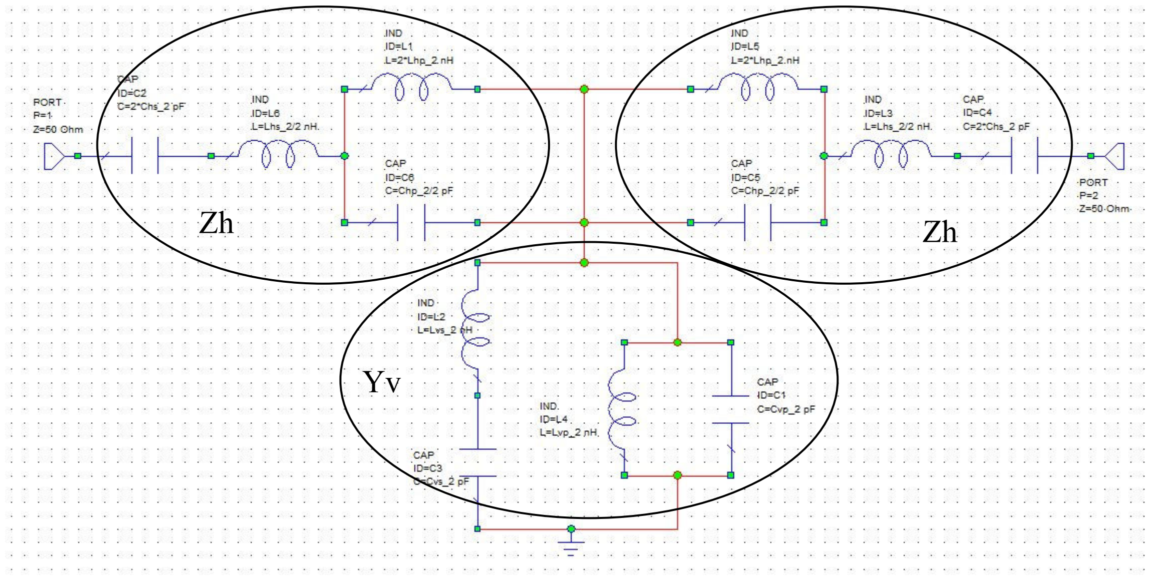


Fig. 1. Generalized NRI-TL circuit Model

In Fig. 2, the circuit of Fig. 1 can be represented by the following 2-port network, which has the ABCD matrix of (1).

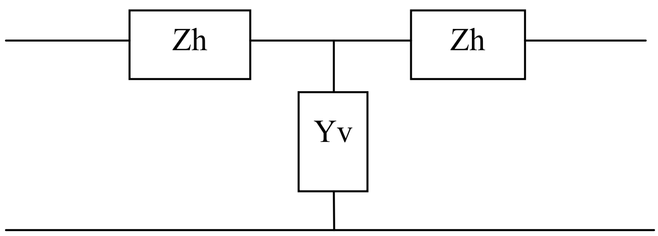


Fig. 2. 2-port network representation of the circuit in Fig.1

The ABCD Matrix of the above 2-port circuit is given by the well known relation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+Z_h Y_v & Z_h(2+Z_h Y_v) \\ Y_v & 1+Z_h Y_v \end{bmatrix} \quad (1)$$

Where in the case of the Generalized NRI-TL unit cell,

$$Z_h = j\omega L_{hs}/2 + \frac{1}{j\omega 2C_{hs}} + \frac{1}{j\omega C_{hp}/2 + \frac{1}{j\omega 2L_{hp}}} \quad (2)$$

And

$$Y_v = j\omega C_{vp} + \frac{1}{j\omega L_{vp}} + \frac{1}{j\omega L_{vs} + \frac{1}{j\omega C_{vs}}} \quad (3)$$

Considering that the resonant frequencies of the resonators in the circuit are described by :

$$\omega_{hs}^2 = 1/\sqrt{L_{hs}C_{hs}} \quad (4)$$

$$\omega_{hp}^2 = 1/\sqrt{L_{hp}C_{hp}} \quad (5)$$

$$\omega_{vp}^2 = 1/\sqrt{L_{vp}C_{vp}} \quad (6)$$

$$\omega_{vs}^2 = 1/\sqrt{L_{vs}C_{vs}} \quad (7)$$

Then, Zh and Yv can be rewritten as

$$Z_h = j\omega L_{hs}/2\left(1 - \frac{\omega_{hs}^2}{\omega^2}\right) - \frac{j}{\omega C_{hp}/2\left(1 - \frac{\omega_{hp}^2}{\omega^2}\right)} \quad (8)$$

And

$$Y_v = j\omega C_{vp}\left(1 - \frac{\omega_{vp}^2}{\omega^2}\right) - \frac{j}{\omega L_{vs}\left(1 - \frac{\omega_{vs}^2}{\omega^2}\right)} \quad (9)$$

From the theory on the analysis of infinite periodic structures, for the case of a nonattenuating, propagating wave on the periodic structure, the corresponding Bloch propagation constant (dispersion relation) of the above unit cell, is given by the following equation:

$$\cos\beta d = \left(\frac{A+D}{2}\right) \quad (10)$$

By substituting (8) and (9) into (10), we obtain the following dispersion relation:

$$\cos\beta d = 1 + Z_h Y_v \quad (11)$$

Under the closed stopband condition, we get two passbands, separated by a stopband centered at ω_{or}, which is the resonant frequency of the parallel resonator in the horizontal branch of the circuit as well as the series resonator in the vertical branch of the circuit. At the center of the passband, the phase velocity is infinite and this can be expressed as the frequencies where β=0. The stopband at ω_{or} is created when the vertical branch becomes short circuited and the horizontal branch becomes open circuited. This means that at the stopband, there is no propagation and this can be expressed as the frequencies where β is infinite. By (11), the β=0 frequencies are found by if we set $Z_h = 0$ and $Y_v = 0$. To find the frequencies where $Z_h = 0$ and $Y_v = 0$, we set (8) and (9) equal to zero, resulting in the following equations:

$$\omega_{hor\_zero} = 0.5(\omega_{hp}^2 + \omega_{hs}^2 + \omega_{hshp}^2) \pm 0.5\sqrt{(\omega_{hp}^2 + \omega_{hs}^2 + \omega_{hshp}^2)^2 - 4\omega_{hs}^2\omega_{hp}^2} \quad (12)$$

Where $\omega_{hshp}^2 = 4 / L_{hs}C_{hp}$ (13)

$$\omega_{ver\_zero} = 0.5(\omega_{vs}^2 + \omega_{vp}^2 + \omega_{vsvp}^2) \pm 0.5\sqrt{(\omega_{vs}^2 + \omega_{vp}^2 + \omega_{vsvp}^2)^2 - 4\omega_{vs}^2\omega_{vp}^2} \quad (14)$$

Where $\omega_{vsvp}^2 = 1 / L_{vs}C_{vp}$ (15)

By setting these two sets of zero frequencies equal, we get the closed stopband condition, when

$\omega_{hp} = \omega_{vs} = \omega_{or}$ (16)

$\omega_{hs} = \omega_{vp}$ (17)

And $\omega_{hshp} = \omega_{vsvp}$ or $\dfrac{L_{hs}C_{hp}}{4} = L_{vs}C_{vp}$ (18)

Equation (18), defines the center of the stopband that separates the two passbands. This can be seen by inspection of Fig. 1, where at the frequency ω_{or} the parallel resonator in the horizontal branch becomes open circuited and the series resonator in the vertical branch becomes short circuited. The passband centers are defined by equation (15), which are defined at the frequencies where β=0.
In addition, as a result of equations (16)-(18), the impedance of the horizontal branch Z_h and the admittance of the vertical branch Y_v become equal within a constant, defined below by the calculation of the Bloch impedance of the circuit.
Again, from the analysis of periodic structures, we know that

$$Z_{Bloch} = \frac{A - D + \sqrt{(A+D)^2 - 4}}{2C} \quad (19)$$

By substitution of the ABDC parameters, we then obtain

$$Z_{Bloch} = \sqrt{\frac{Z_h}{Y_v}}\sqrt{2 + Z_{hY_v}} \quad (20)$$

At around the β=0 frequencies, (20) becomes

$$Z_{Bloch} \approx \sqrt{\frac{2Z_h}{Y_v}} \quad (21)$$

$$\Rightarrow 2Z_h = Z_{Bloch}Y_v \, (22)$$

$$\Rightarrow 2Z_h = qY_v \, (23)$$

Where $q = Z_{Bloch}$ is the Bloch impedance at the centers of the passbands where β=0. Substituting (8) and (9) into (21), results in the following expression:

$$Z_{Bloch} \approx \sqrt{\frac{L_{hs}}{C_{vp}}} \, (24)$$

From (11) and (23),

With $A^2 = -\dfrac{2(1-\cos\beta d)}{(q)^2} = -Y_{vp}{}^2$ (25)

From (3) and (23), the following polynomial is established:

$$\omega^4 - \frac{A}{C_{vp}}\omega^3 - (\omega_{or}^2 + \omega_{vp}^2 + \omega_{vsvp}^2)\omega^2 + \frac{A\omega_{or}^2}{C_{vp}}\omega + \omega_{or}^2\omega_{vp}^2 = 0 \, (26)$$

The above polynomial has the four roots, $-\omega_1$, ω_2, $-\omega_3$, ω_4. Therefore, the polynomial yields four equations for the four unknowns C_{vp}, ω_{or}, ω_{vp}, ω_{vsvp}.

By relating the known roots to the polynomial coefficients, we get:

$$C_0 = \omega_1\omega_2\omega_3\omega_4 = \omega_{or}^2\omega_{vp}^2 \, (27)$$

$$C_1 = (-\omega_1 + \omega_2 - \omega_3 + \omega_4) = \frac{A}{C_{vp}} \, (28)$$

$$C_2 = (\omega_1\omega_2 - \omega_1\omega_3 + \omega_1\omega_4 + \omega_2\omega_3 - \omega_2\omega_4 + \omega_3\omega_4 = (\omega_{or}^2 + \omega_{vp}^2 + \omega_{vsvp}^2) \, (29)$$

$$C_3 = (-\omega_1\omega_3\omega_4 + \omega_2\omega_3\omega_4 - \omega_1\omega_2\omega_3 + \omega_1\omega_2\omega_4) = \frac{A\omega_{or}^2}{C_{vp}} \, (30)$$

From the above four equations, the four unknowns can be solved.

$$C_{vp} = \frac{A}{C_1} \, (31)$$

$$\omega_{or}^2 = \frac{C_3}{C_1} \quad (32)$$

$$\omega_{vp}^2 = C_0 \frac{C_1}{C_3} \quad (33)$$

$$\omega_{vsvp}^2 = C_2 - \frac{C_3}{C_1} - C_0 \frac{C_1}{C_3} \quad (34)$$

Then, the rest of the component values can be evaluated.

$$L_{vp} = \frac{1}{\omega_{vp}^2 C_{vp}} \quad (35)$$

$$L_{vs} = \frac{1}{\omega_{vsvp}^2 C_{vp}} \quad (36)$$

$$C_{vs} = \frac{1}{\omega_{or}^2 L_{vs}} \quad (37)$$

$$L_{hs} = q C_{vp} \quad (38)$$

$$C_{hs} = \frac{1}{\omega_{vp}^2 L_{hs}} \quad (39)$$

$$C_{hp} = \frac{4 L_{vs} C_{vp}}{L_{hs}} \quad (40)$$

$$L_{hp} = \frac{1}{\omega_{or}^2 C_{hp}} \quad (41)$$

Theory of a GNRI-TL with host TL

Any such realizable unit cell such as the one discussed above, must unavoidably have transmission line sections to host the lumped element components that make up the unit cell. The effect of these transmission lines is a shift of the target frequencies at a desired phase shift. Thus a new model must be considered, where these effects are calculated and taken into account when designing such unit cells.

One way to calculate these effects is by following the same reasoning as above, but the unit cell model must also include the transmission lines. Fig. 3 shows how this is done.

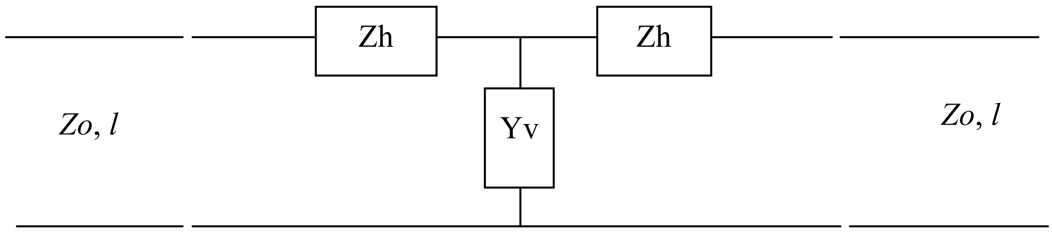


Fig. 3. Unit Cell 2-port with host TL

The ABCD Matrix of the above 2-port circuit is given by the relation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} 1+Z_hY_v & Z_h(2+Z_hY_v) \\ Y_v & 1+Z_hY_v \end{bmatrix} \begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{bmatrix} \quad (42)$$

After evaluating the above matrix multiplication, the following ABCD parameters result:

$$A = (1+Z_hY_v)\cos^2\beta l + jZ_0Y_v[Z_h(1+Z_hY_v)+Z_h]\sin\beta l\cos\beta l + jY_0\sin\beta l\cos\beta l - Y_0Z_0(1+Z_hY_v)\sin^2\beta l = D$$

where l is the length of the host transmission line and βl represents the phase shift incurred by the transmission line.

Thus, the new dispersion relation, becomes:

$$\cos\beta d = (1+Z_hY_v)(\cos^2\beta l - \sin^2\beta l) + j\{Y_o[Z_h(1+Z_hY_v)+Z_h]\}$$

The imaginary term of the above equation must be zero, which results in the following:

$$\cos\beta d = (1+Z_hY_v)(2\cos^2\beta l - 1)$$

As an example, let's consider the case where a phase shift $\Phi = \beta d = 45^0$ is required at the frequencies f1=1GHz, f2=2GHz, f3=4GHz and f5=5GHz. The host transmission line, has a phase shift of 0.25 radians (around 15 degrees). Plotting the above two dispersion relations with respect to frequency, results in the following graph in Fig.4.

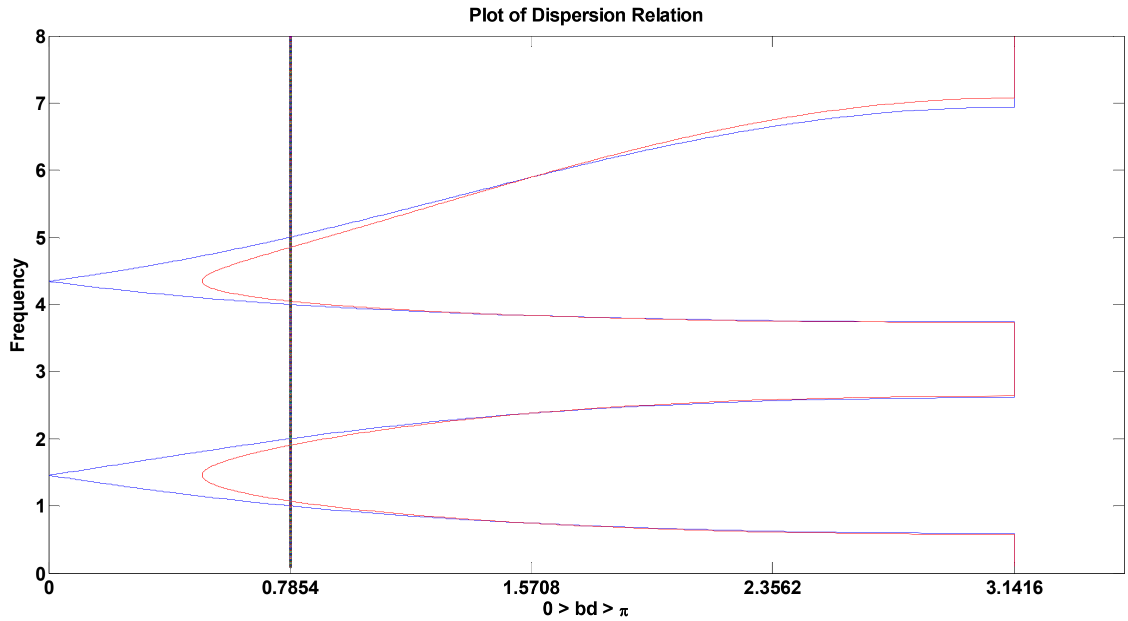
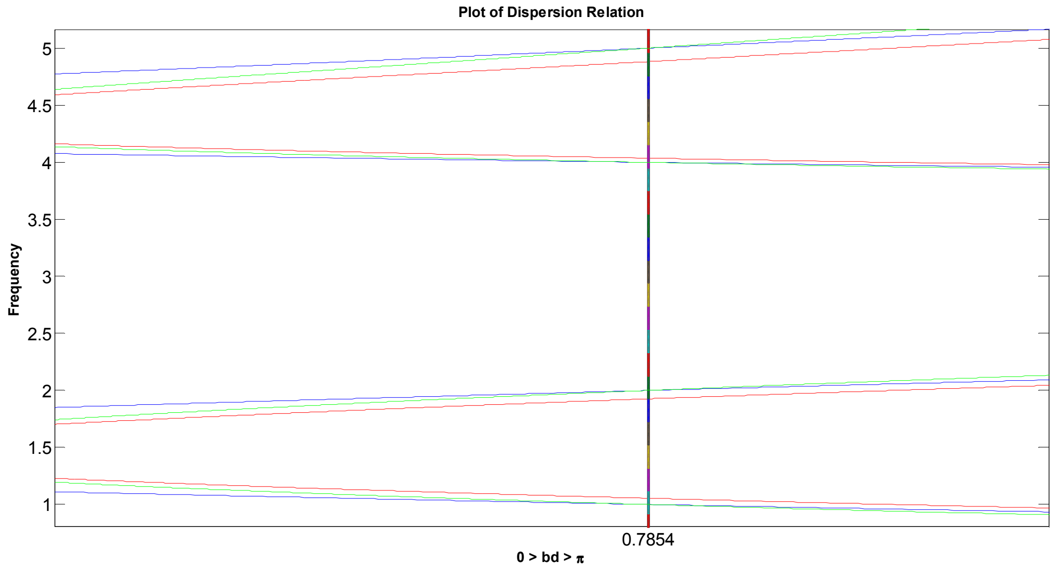


Fig. 4. Plot of the dispersion relations, with and without the host TL

The blue line represents the Generalized NRI-TL and the red line represents the Generalized NRI-TL with the host TL. The black vertical line is located at βd=45 degrees. The four points where the βd=45 degree line cuts the blue dispersion line, are the four design frequencies. It can be seen that this βd=45 degree line intersects the red dispersion line not at the design frequencies, but at frequencies that are offset by a certain amount. This is the error that should be rectified.

To do this, the series inductance and the shunt capacitance of the host TL should be calculated and subtracted from the Generalized NRI-TL values of L_{hs} and C_{hp} respectively. Then a recalculation of all the other component values must be done, and the unit cell performance recalculated. Fig. 5 below, shows the correction that occurs and is represented by the green curve which now coincides with the blue curve of the original, theoretical unit cell at the four design frequencies and at the desired phase shift of 45 degrees.



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